TOPIC 1 OVERVIEW

Operating with Rational Numbers

How are the key concepts of *Operating with Rational Numbers* organized?

In *Operating with Rational Numbers*, the focus is on building fluency when operating with positive and negative rational numbers. Students begin this topic by applying their knowledge of adding and subtracting positive and negative integers to the set of rational numbers. Next, students divide integers, resulting in rational numbers, given the divisor is not 0. They learn that the decimal form of quotients of integers — i.e., rational numbers — always repeat or terminate. Students recognize that the placement of the negative sign on a rational number does not matter $\left(\text{e.g., } -\frac{p}{q} = \frac{-p}{q} = \frac{p}{-q}\right)$.

Add & subtract positive & negative integers

Students then apply the rules for multiplying and dividing integers to the set of rational numbers. Finally, students apply familiar properties (additive inverse, distributive property, etc.) to rational numbers. They practice expanding and factoring expressions involving -1 and recognize subtraction as applying the additive inverse.

Apply rules for mult & dividing integers (Add inverse)

Students represent variable expressions on a number line and making connections between variable and numeric expressions. They use their previous knowledge of evaluating expressions to verify their reasoning. Students then apply the distributive property as a strategy to write equivalent expressions and factor linear expressions in a variety of ways.

dist. property & writing EQ fractions

Repres #1

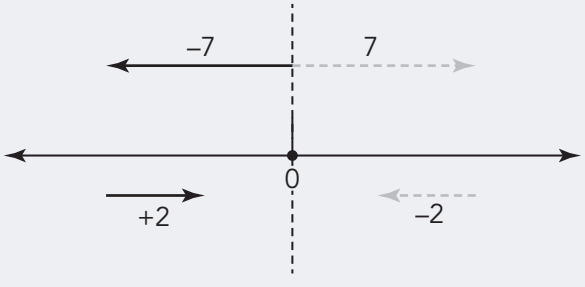
Math Representation

Consider the expression $-7 + 2$.

When you reflect the model of $-7 + 2$ across 0 on the number line, the result is $7 - 2$.

So, $(-7 + 2)$ is the opposite of $(7 - 2)$.

This means that $-7 + 2 = -(7 - 2)$.



What is the entry point for students?

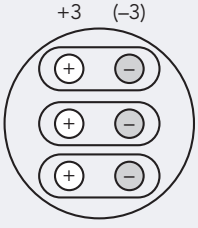
In previous courses, students represented integer operations with concrete models and connected the actions with the models to standardized algorithms. Then, they worked on building fluency in using all four operations with integers. In this course, they will build off of the foundation developed around operating with positive rational numbers and operating with integers, to now building fluency in operating with the full set of rational numbers.

This topic combines students' knowledge of expressions and negative numbers on a number line to develop number line models for variable expressions.

Math Representation

You can model the addition of integers using two-color counters that represent positive (+) charges and negative (−) charges.

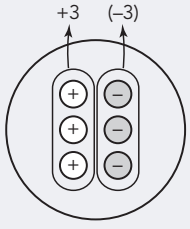
The expression 3 + (−3) can be modeled in different ways



Each positive charge is paired with a negative charge.

Each pair of positive and negative charges has no charge.

3 + (−3) = 0



Three positive charges and three negative charges have no charge.

3 + (−3) = 0

Why is *Operating with Rational Numbers* important?

Rational numbers are the primary set of numbers with which students will work in their educational and non-educational lives. It is essential that students develop a strong conceptual foundation for operating with rational numbers to provide the foundation for manipulating and representing increasingly complex numeric and algebraic expressions. In future courses, students will focus more on expressions and equations than on numbers, including rational expressions, equations, and functions. The work in this topic supports students as they develop automaticity in operating with rational numbers which is a key skill for engaging with more complex mathematical content.

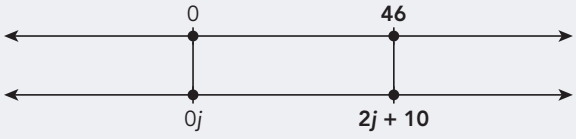
Visualizing simple variable expressions on a number line will carry through the entire topic to help students develop a concrete idea relating expressions to each other and operating with algebraic expressions. In this course, students will extend previous understanding around operating with positive rational numbers and integers to now build fluency in operating with the full set of rational numbers.

Repres #3

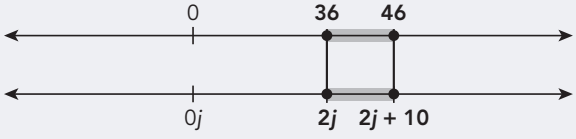
Math Representation

You can use double number lines to help you solve equations. When solving an equation, equality must be maintained. What is done to one expression must be done to the equivalent expression to maintain equality.

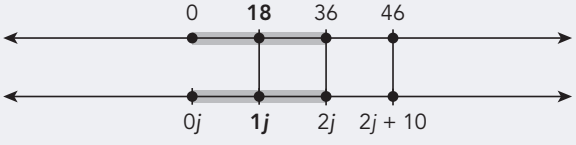
For example, solve the equation $2j + 10 = 46$. First, draw a model to set up the equation.



Next, start decomposing the variable expression. Place $2j$ in relationship to $2j + 10$. The expression $2j$ is 10 to the left of $2j + 10$. To maintain equality, place a number 10 to the left of 46. So, $2j = 36$.



The expression $1j$, or j, is halfway between $0j$ and $2j$, and 18 is halfway between 0 and 36. So, $j = 18$.



How does a student demonstrate understanding?

Students will demonstrate understanding of the standards in this topic when they can:

- Explain why the quotient of two integers, except when the divisor is 0, is always a rational number.
- Apply properties of operations to calculate numbers in any form and convert between numeric forms when necessary.
- Solve multi-step real-world and mathematical problems with numbers in any form (whole numbers, fractions, and decimals), using tools strategically.
- Solve real-world problems by adding, subtracting, multiplying, and dividing rational numbers.
- Use properties of operations to add, subtract, multiply, and divide rational numbers.

NEW KEY TERMS

- percent error [error porcentual]
- variable [variable]
- algebraic expression [expresión algebraica]
- linear expression [expresión lineal]
- constraint
- evaluate an algebraic expression [evaluar una expresión algebraica]
- factor [factor]
- coefficient [coeficiente]
- common factor [factor común]
- greatest common factor (GCF) [máximo común factor/divisor]

- Evaluate algebraic expressions with rational coefficients.
- Use properties of operations to write equivalent expressions.
- Factor and expand linear expressions with rational coefficients using the distributive property.

How do the activities in *Operating with Rational Numbers* promote student expertise in the TEKS mathematical process standards?

Every topic is written with the goal of creating mathematical thinkers who are active participants in class discourse, so elements of the TEKS mathematical process standards should be evident in all lessons. Students are expected to make sense of problems and work towards solutions, reason using concrete and abstract ideas, and communicate their thinking while providing a critical ear to the thinking of others.

Students are expected to recognize that familiar properties can be applied to rational numbers and to develop precision with the rules as they apply them to the set of rational numbers (7.1F). Algebraic expressions exhibit structure and can be composed and decomposed to reveal that structure or to help learners attend to a particular aspect of that structure. Students are expected to develop precision as they achieve proficiency with the skills of creating and evaluating equivalent expressions with rational coefficients (7.1G). Throughout the topic, they are expected to use number sense to determine whether their answers are reasonable (7.1D).

How can you use cognates to support EB students?

Cognates are provided for new key terms when applicable. Guide students in exploring spelling patterns across cognate pairs, illustrating how English word endings like *-ate*, *-ent*, *-tion*, and *-sion* correspond with Spanish *-ar*, *-ente*, *-ción*, and *-sión*. Point out that double consonants in English often appear as a single consonant in Spanish. Examples of these cognate relationships include *evaluate* with *evaluar*, *coefficient* with *coeficiente*, *notation* with *notación*, and *expression* with *expresión*. As students become adept at recognizing these patterns, they can more easily link new language and vocabulary with their prior knowledge.

MODULE 3, TOPIC 1 PACING GUIDE

165-Day Pacing

3 Reasoning Algebraically

TOPIC 1: Operating with Rational Numbers

1 DAY PACING = 45-MINUTE SESSION

TEKS Mathematical Process Standards: 7.1A, 7.1B, 7.1C, 7.1D, 7.1E, 7.1F, 7.1G

ELPS: 1.D, 1.E, 1.F, 2.D, 2.G, 2.H, 3.A, 3.B, 3.D, 3.G, 4.A, 4.C, 4.K, 5.E

Topic Pacing: 17 Days

Lesson	Lesson Title	Highlights	TEKS*	Pacing
1	Adding and Subtracting Rational Numbers	Students apply their knowledge of adding and subtracting positive and negative integers to a set of rational numbers. **Materials Needed:** Problem-Solving Model Graphic Organizer	7.2A 7.3A **7.3B**	2
2	Quotients of Integers	Students divide integers. They learn that the quotients of any two integers are rational numbers. Students express rational numbers written as negative fractions in equivalent forms by changing the negative sign's position. Finally, they perform operations with positive and negative rational numbers to solve real-world problems. **Materials Needed:** Problem-Solving Model Graphic Organizer	7.3A **7.3B**	2
3	Simplifying Expressions to Solve Problems	Students solve real-world problems involving simplifying numeric expressions using the four operations and signed rational numbers. Students also evaluate expressions with signed rational numbers for a variable and use the order of operations to simplify. **Materials Needed:** Problem-Solving Model Graphic Organizer	7.3A **7.3B** **7.4D**	2
4	Using Number Properties to Interpret Expressions with Signed Numbers	Students solve mathematical problems involving simplifying numeric expressions using number properties and signed rational numbers. Students will also use what they know about the opposites of numbers to derive a method for distributing and factoring with -1 and to convert subtraction to the addition of the opposite of a number. **Materials Needed:** None	7.3A **7.3B** **7.4D**	2

*Bold TEKS = Readiness Standard

MODULE 3, TOPIC 1 PACING GUIDE

165-Day Pacing

Lesson	Lesson Title	Highlights	TEKS*	Pacing
5	**Evaluating Algebraic Expressions**	Students review variables, algebraic expressions, and how to evaluate algebraic expressions. They plot a variety of variable expressions with x on a number line, first under the condition that $x > 0$, and then under the condition that $x < 0$, focusing on the distance of x from 0 to determine the placement of the expressions. Students substitute values for the variable to validate the correct placement of the expressions on the number lines. They then substitute values for unknowns in two related contexts. Finally, students formally review evaluating an algebraic expression and practice this skill, with and without tables. **Materials Needed:** Index Cards, Tape	7.3A **7.3B**	1
6	**Rewriting Expressions Using the Distributive Property**	Students rewrite linear expressions using the distributive property. First, they plot related algebraic expressions on a number line by reasoning about magnitude. Students realize that rewriting the expressions reveals structural similarities in the expressions, which allows them to more accurately plot the expressions. They then review the distributive property. Students expand algebraic expressions using both the area model and symbolic representations, focusing on the symbolic. They then reverse the process to factor linear expressions. Students factor expressions by factoring out the greatest common factor and by factoring out the coefficient of the linear variable. Finally, students rewrite expressions in multiple ways by factoring the same value from each term of the expression. **Materials Needed:** None	7.3A **7.3B**	2
End of Topic Assessment				1
Learning Individually with Skills Practice *Schedule these days strategically throughout the topic to support student learning.*				5

*Bold TEKS = Readiness Standard

MODULE 3, TOPIC 1 PACING GUIDE

~~165-Day Pacing~~

150 Day Pacing

1 DAY PACING = 45-MINUTE SESSION

Day 1	Day 2	Day 3	Day 4	Day 5
TEKS: 7.2A, 7.3A, **7.3B**			TEKS: 7.3A, **7.3B**	
LESSON 1 Adding and Subtracting Rational Numbers GETTING STARTED ACTIVITY 1	**LESSON 1** continued ACTIVITY 2 ACTIVITY 3 TALK THE TALK	**LEARNING INDIVIDUALLY** Skills Practice This is a suggested placement. *Move based on student data and individual needs.*	**LESSON 2** Quotients of Integers GETTING STARTED ACTIVITY 1	**LESSON 2** continued ACTIVITY 2 TALK THE TALK
Day 6	**Day 7**	**Day 8**	**Day 9**	**Day 10**
	TEKS: 7.3A, **7.3B, 7.4D**			TEKS: 7.3A, **7.3B**
LEARNING INDIVIDUALLY Skills Practice This is a suggested placement. *Move based on student data and individual needs.*	**LESSON 3** Simplifying Expressions to Solve Problems GETTING STARTED ACTIVITY 1	**LESSON 3** continued ACTIVITY 2 ACTIVITY 3 TALK THE TALK	**LEARNING INDIVIDUALLY** Skills Practice This is a suggested placement. *Move based on student data and individual needs.*	**LESSON 4** Using Number Properties to Interpret Expressions with Signed Numbers GETTING STARTED ACTIVITY 1
Day 11	**Day 12**	**Day 13**	**Day 14**	**Day 15**
		TEKS: 7.3A, **7.3B**	TEKS: 7.3A, **7.3B**	
LESSON 4 continued ACTIVITY 2 ACTIVITY 3 TALK THE TALK	**LEARNING INDIVIDUALLY** Skills Practice This is a suggested placement. *Move based on student data and individual needs.*	**LESSON 5** Evaluating Algebraic Expressions GETTING STARTED ACTIVITY 1 ACTIVITY 2 ACTIVITY 3 TALK THE TALK	**LESSON 6** Rewriting Expressions Using the Distributive Property GETTING STARTED ACTIVITY 1 ACTIVITY 2	**LESSON 6** continued ACTIVITY 3 TALK THE TALK
Day 16	**Day 17**			
LEARNING INDIVIDUALLY Skills Practice This is a suggested placement. *Move based on student data and individual needs.*	**END OF TOPIC ASSESSMENT**			

*Bold TEKS = Readiness Standard

Skills practice is flexible

spaced practice = spiral practice

interleaved prac = more than 1 type of problem or skill set.

use st. data to select "must do" problems

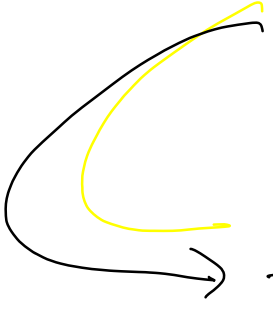
How can you incorporate Skills Practice with students?

There are five Learning Individually days scheduled within this topic. The placement of these days within the topic is flexible. The intent is to distribute spaced and interleaved practice throughout a topic and throughout the year. It is not necessary for students to complete all Skills Practice for the topic and different students may complete different problem sets. You should use data to strategically assign problem sets aligned to individual student needs. You should analyze student responses from the following embedded assessment opportunities to help assess individual needs: Essential Questions, Talk the Talks, Student Self-Reflections, and End of Topic Assessments. For students who are building their proficiency, you can assign problem sets to target specific skills. For students who have demonstrated proficiency, there are extension problems of varied levels of challenge.

extension probs to challenge sts

How can you identify whether students are ready for new learning?

The Prepare section of the Lesson Assignments and the Spaced Practice sets of Skills Practice can serve as diagnostic tools. Depending on available time, you can assign the Prepare section of the Lesson Assignments as homework or as a warm-up to identify students' prior knowledge for the upcoming lesson's activities. You can also use the Spaced Practice sets of Skills Practice to analyze individual students' level of proficiency on standards from previous topics.



prepare section of the assnment can serve as a warm up or prework.